# Optimum Implementation of the Polynomials in Coordinate Transformation Process

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عند إجراء الأعمال المساحية في مصر باستخدام الأقمار الصناعية فان مشكلة التحويل بين سطح الإسناد الدولي وسطح الإسناد المحلى تعتبر مشكلة من أهم المشكلات آلتي تقابل العاملين في هدا المجال ودلك للحصول على افضل عناصر التحويل بين سطحي الإسناد. وقد تمت دراسات عديدة للحصول على عناصر التحويل باستخدام ارصاد ونماذج رياضية مختلفة وبسبب قلة البيانات المتوفرة (أحداثيات النقاط في كل من سطحي الإسناد) تم تطبيق كثيرات الحدود وغالبا بدون نقاط اختبار لها. لدلك نجد أن كثيرة الحدود المستخدمة ( آلتي تم استنتاجها من نقاط الحل) تعطى اقل فروق عند النقاط آلتي تم من خلالها الحصول على معاملات كثيرة الحدود.لدلك يقوم البحث بدراسة سلوك كثيرة حدود تعطى افضل نتائج عند نقاط الحل وايضا عند نقاط الاختبار مع معرفة تأثير عدد نقاط الحل والمسافة بين هده النقاط على دقة كثيرة الحدود وايضا مقارنة دقة النتائج بين استخدام عناصر التحويل وكثيرة الحدود المستخدمة.

## Abstract

**In Egypt, different trials were done to obtain the best set of transformation parameters between the Egyptian Datum (ED) and the World Geodetic System 1984 (WGS-84). They differed in their results because of the differences in the used data and models of transformation. Different mathematical models are used, polynomials are among the used models. Because of the lack of the available common data points, some of the researchers applied the polynomials without check points. Normally Polynomials show good behavior, minimum residuals, at the data points and there is no chance to be examined at enough check points. Therefore, users can be tricked while using polynomials in such aim. In this research, the best polynomial from previous research is chosen to be examined using data and check points. So the goal here is not to obtain transformation parameters but to investigate the behavior of the polynomial against the number of used data points and the average spacing between those data points.**

## 1- Introduction

Transforming the results of the GPS into the local datum is investigated by many researchers in Egypt. Some of those investigations are summarized as follows:

- Bekheet, 1993 applied two transformation models namely Bursa model and first order polynomials in two dimensions. He applied the two models to define relationships among the three datums ED, WGS72, WGS84. He used 8 common points in the solution and there were no check points to assess the solutions. The results of the used polynomials were better than the results of Bursa model at the used data points.
- Abd-Elmotaal, 1994 presented the comparison of polynomial and

similarity transformation based datum shifts for Egypt. He used 8 common points from first order geodetic stations known in both WGS84 and Egyptian Datum as data points. These points are located in Egyptian Eastern Desert, and thier WGS84 coordinates have been taken from [Finnmap 1989] and no check points are used. The results for both similarity transformation by using BURSA model and the coefficients of the surface polynomial second order showed that the polynomial is better than Bursa model.

- El-Tokhey, 1999 computed transformation parameters between the Egyptian Datum and WGS-84 by using two models (BURSA model and two dimensional surface polynomials second order). He used 15 common points from the Egyptian Survey Authority (ESA) project of the High Accuracy Referance Network (HARN) as data points. The known coordinates of these points in the Egyptian Datum were taken from the final adjusted coordinates of [Awad, 1997]. The derived transformation parameters have been checked at 16 stations of the Egyptian Aviation Authority (EAA). The coordinates of these stations are known in both WGS84 and the Egyptian Datums. Finally [EL-Tokhey, 1999] concluded that the used polynomials are better than Bursa model.

- Gomaa and Alnaggar, 2000 presented the geodetic datum transformation techniques for GPS surveys in Egypt by using two groups (similarity models Bursa and Molodensky) and (two dimensional surface polynomials multiple regression). In the first group, three, four, and seven parameters are computed for each model. But in the second group the procedure is to add one variable at a time to the equation. The available geodetic coordinates were19 first order geodetic stations known in both WGS84 and ED and 15 common points were used as solution points. The GPS coordinates came from two sources: (HARN) and the remaining stations have been observed by the Survey Research Institute (SRI) as part of the Egyptian National Standardization Gravity Network (ENSGN97). Four stations have been considered as check points. The results showed that multiple regression is better than Bursa and Molodensky according to [Gomma and Alnaggar, 2000].

- Shaker et al, 2003 compared several mathematical models of coordinate transformations. They investigated 12 models including different kinds of polynomials. The main conclusion was that the polynomials have better results at the data points but not necessarily at the check points. The idea of this research is extracted from this conclusion. Optimum data points and optimum spacing among the data points are investigated for the optimum implementation of the used polynomial in transformation process. The best polynomial in 2D from [Ibid.] is adopted in this research.

## 2- Data Used

The data used in this research are composed of:

- Eighteen first order triangulation points have local coordinates and WGS-84 coordinates through GPS observations. They are taken from [Saad, 1998]. Thirteen of them are used as data points in the different solutions and named here the actual data points. The other five are used as check points to estimate the quality of the solutions and named actual check points, Figure (1).
- Seventeen simulated points are taken as a grid covers the whole territory of Egypt. Their coordinates are assumed regularly to form a grid and their WGS-84 coordinates are assigned to them. They are named simulated grid data points. Their description is found in subsection (3-3)
- Eighteen simulated points are taken randomly inside the grid covers the whole territory of Egypt. They have assumed WGS-84 coordinates. They are named simulated grid check points, Figure (2).
- A reliable seven transformation parameters for Egypt, taken from [Saad, et al, 1998] are used to obtain corresponding local coordinates for the points in the above two items. Thirty one common points were available, 15 points were used in the solution and 10 points were used to check the solution. 6 points were rejected. The residuals of the solution at the data points ranged from 0.11 to 1.43 m with mean value 0.48 m and St dv

0.37 m. the residuals at 10 check points ranged from 0.12 to 1.63 m with mean value 0.89 m and Stdv 0.64 m.

- The used transformation model is the well known Molodenski model.

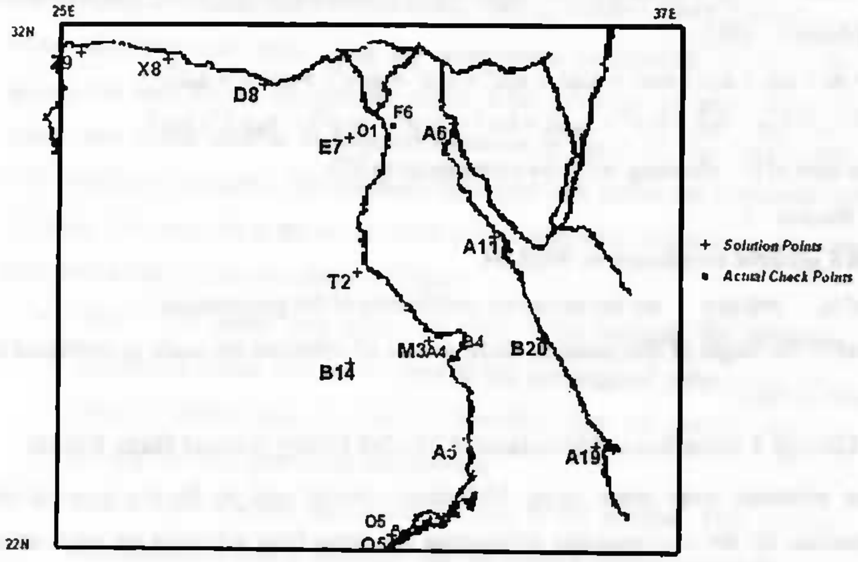


Figure (1) Actual data and check points.

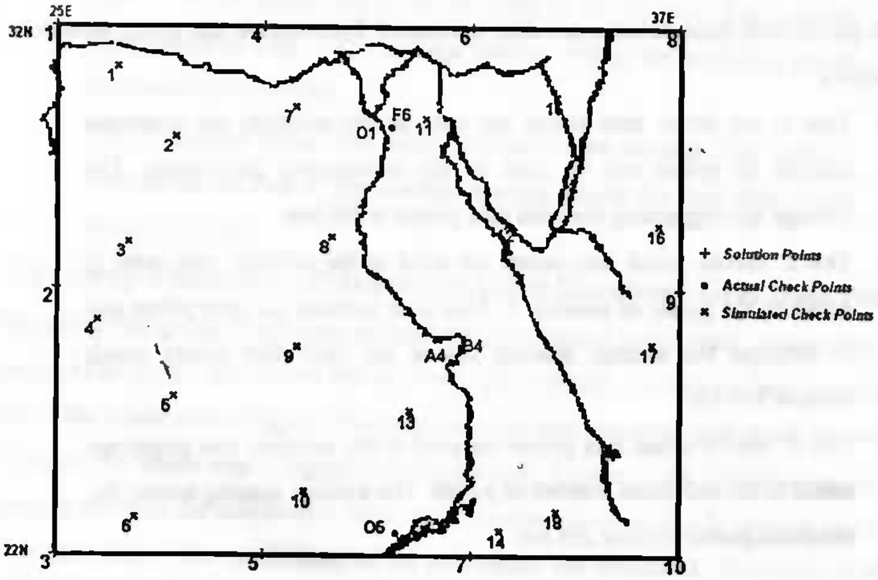


Figure (2) The Ten grid data and check points.

## 3- Methodology

Two mathematical models are utilized in the computations, namely Molodenski 7 parameters model and the adopted 2D polynomials. Details of Molodenski can be found in [Nassar, 1994]. The adopted 2D polynomial is used up to the third order [Anderson and Mikhail, 1998]:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + a_6\varphi^3 + a_7\varphi^2\lambda + a_8\varphi\lambda^2 + a_9\lambda^3$$

$$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3\varphi^2 + b_4\varphi\lambda + b_5\lambda^2 + b_6\varphi^3 + b_7\varphi^2\lambda + b_8\varphi\lambda^2 + b_9\lambda^3$$

In the case of transforming WGS-84 coordinates to ED;

$\Delta\varphi = \varphi_{WGS-84} - \varphi_{ED}$

$\varphi$ and $\lambda$ geodetic coordinates in WGS-84

$a_i$ and $b_i$   i=0 to n   are the unknown coefficients of the polynomials.

To verify the target of this research, three groups of solutions are made as explained in the next sections:

### 3-1 Group 1 Solutions: Molodenski Model Using Actual Data Points

Those solutions were made using Molodenski model and to be the base of the comparison for the corresponding polynomial solutions. Four solutions are made using actual data points. The transformation parameters are computed. The residuals at the solution actual data points are computed with their statistics. The residuals at the actual check points with their statistics are also computed. Four tests at this group were done as follows:

- Test 1: ten actual data points are used in the solution, the minimum number of points can be used in the investigated polynomial. The average spacing among the used data points is 390 km.
- Test 2: eleven actual data points are used in the solution, one point is added to the points of solution 1. This is to increase the data points and to decrease the average spacing among the used data points which became 355 km.
- Test 3: twelve actual data points are used in the solution, two points are added to the minimum number of points. The average spacing among the used data points became 295 km.

- Test 4: thirteen actual data points are used in the solution, three points are added to the minimum number of points. The average spacing among the used data points became 280 km.

### 3-2 Group 2 Solutions: Polynomials Using Actual Data Points

Those solutions were made using the investigated polynomials to check its results against the well known Molodenski model. Four solutions are made using the same actual data points used in Molodenski solutions in group 1. The coefficients of the polynomial are computed. The residuals for actual data points are computed with their statistics. The residuals at the actual check points with their statistics are also computed. Four tests at this group were done as follows:

- Test 5: ten actual data points are used in the solution, the minimum number of points that can be used in the investigated polynomial. This solution corresponds to Test 1. Recalling that the average spacing among the used data points was 390 km.
- Test 6: eleven actual data points are used in the solution. This solution corresponds to Test 2. Recalling that the average spacing among the used data points is decreased to be 355 km.
- Test 7: twelve actual data points are used in the solution. The solution corresponds to Test 3. The average spacing among the used data points decreased to be 295 km.
- Test 8: thirteen actual data points are used in the solution. The solution corresponds to Test 4. The average spacing among the used data points became 280 km.

### 3-3 Group 3 Solutions: Polynomials Using Simulated Grid Data Points

Recalling that group 2 solutions utilized the investigated polynomial using actual data points. The actual data points are scattered, not regularly distributed, and they do not cover the whole area of Egypt. Therefore the seventeen simulated grid points are created to cover the whole area of Egypt with regular grid, Figures (2:6). This assumption will help in studying the polynomial behavior in the case of using regular spacing and using free error data. The coefficients of the polynomial are computed. The residuals at the solution actual data points are computed with their statistics. The residuals at the actual check points with their statistics are also computed. In this group of solutions, the residuals at simulated check points, scattered over the area of Egypt, are computed because the used actual check points are existed in limited area in Egypt.

- Test 9: ten simulated data points are used in the solution, points from number 1 to number 10, Figure (2). The average spacing among the used data points is 765 km.
- Test 10: eleven simulated data points are used in the solution, points from number 1 to number 11, Figure (3). The average spacing among the used data points is decreased to be 575 km.
- Test 11: twelve simulated data points are used in the solution, points from 1 to 10 and number 12 and 13, Figure (4). The average spacing among the used data points decreased to be 525 km.
- Test 12: thirteen simulated data points are used in the solution, points from 1 to 11 and number 14 and 15, Figure (5). The average spacing among the used data points became 485 km.
- Test 13: fourteen simulated data points are used in the solution, points from 1 to 10 and from number 14 to 17, Figure (6). The average spacing among the used data points became 450 km.

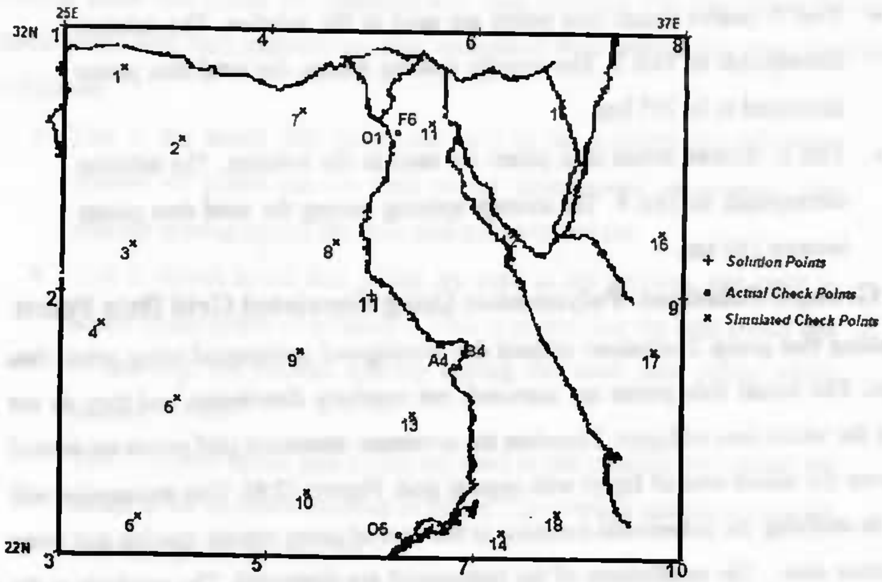


Figure (3) Eleven grid data points and check points.

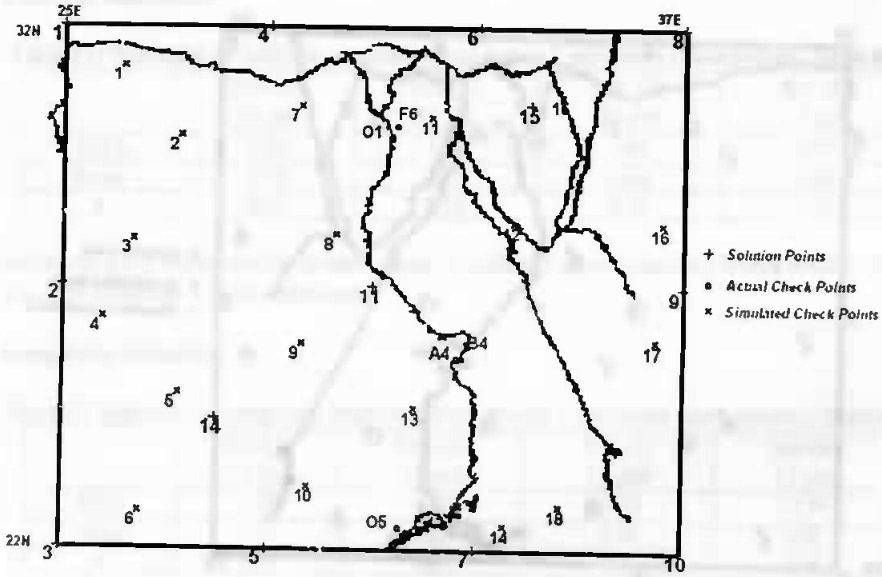


Figure (4) Twelve grid data points and check points.

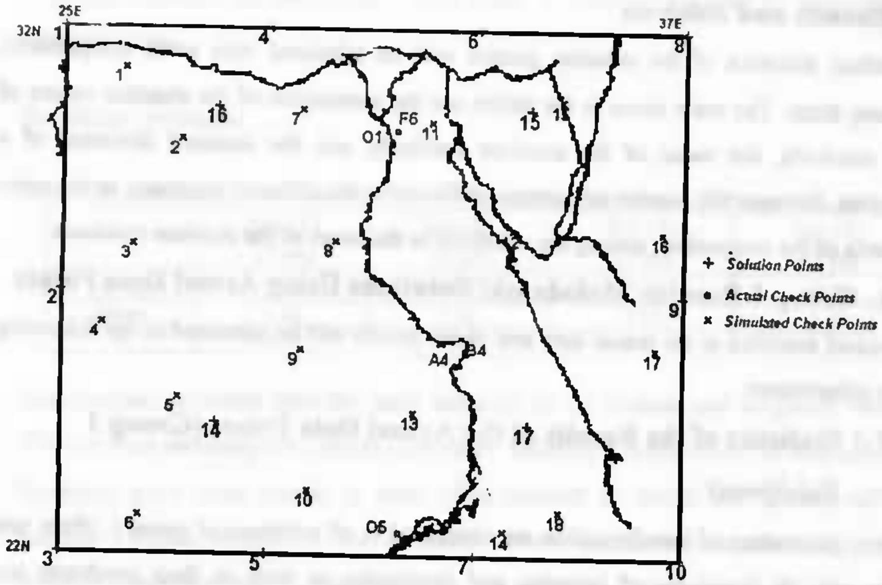


Figure (5) Thirteen grid data points and check points.

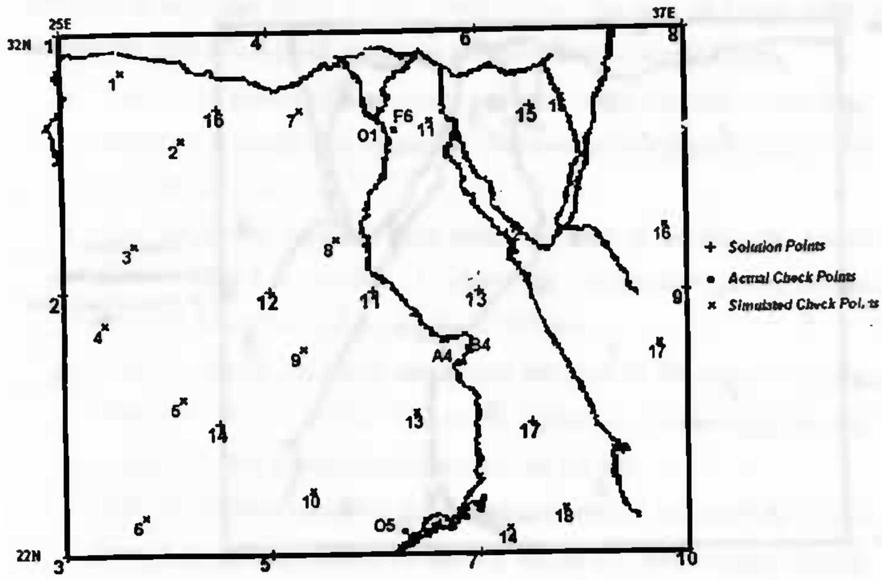


Figure (6) Fourteen grid data points and check points.

## 4- Results and Analysis

Residual statistics of the solution groups will be tabulated with some comparisons among them. The main items in the tables are the summation of the absolute values of the residuals, the mean of the absolute residuals, and the standard deviation of a residual. Because the number of points is different in the different solutions, so the main criteria of the comparison among the solutions is the mean of the absolute residuals.

### 4-1- Group 1 Results: Molodenski Solutions Using Actual Data Points

Residual statistics at the actual data and check points will be tabulated in the following two subsections:

#### 4-1-1 Statistics of the Results at the Actual Data Points (Group 1 Solutions)

Seven parameters of transformation are computed in all solutions of group 1, (from test 1 to test 4). Residuals of latitudes and longitudes as well as their resultants are computed at the data points. The resultant of the shift in latitude and longitude is computed as (sqrt($\Delta\emptyset^2+\Delta\lambda^2$)). Statistics of those residuals are also computed and tabulated as follows:

**Latitude statistics:**

Table (1) Statistics of latitude residuals from group 1 solutions (data points), units in m.

| | Sol.(1) 10 pts | Sol.(2) 11 pts | Sol.(3) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 7.36 | 7.92 | 7.88 | 9.07 |
| Mean | 0.73 | 0.72 | 0.66 | 0.69 |
| σ | 0.56 | 0.54 | 0.56 | 0.55 |

Solutions gave close results to each other, Solution 3 gave relatively better results. The mean of Solution 3 is the minimum.

**Longitude statistics:**

Table(2) Statistics of longitude residuals from group 1 solutions (data points), units in m

| | Sol.(1) 10 pts | Sol.(2) 11 pts | Sol.(3) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 11.09 | 11.14 | 11.76 | 12.99 |
| Mean | 1.11 | 1.01 | 0.98 | 0.99 |
| σ | 0.83 | 0.8 | 0.8 | 0.86 |

The longitude residuals almost have the same behavior of the latitude residuals. The longitude residuals are very close to each other. Solution 3 is a little bit better than the other solutions.

**Resultant residuals:**

Table (3) Statistics of residual resultants from group 1 solutions (data points), units in m

| | Sol.(1) 10 pts | Sol.(2) 11 pts | Sol.(3) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 14.6 | 14.7 | 15.47 | 17.26 |
| Mean | 1.46 | 1.33 | 1.28 | 1.32 |
| σ | 0.79 | 0.48 | 0.81 | 0.76 |

The resultant residuals have the same behavior of the latitude and longitude residuals. The resultant residuals are close to each other. Solution 3 is relatively the best solution. Solutions gave close results to each other because 10 points are already sufficient number of data points for the solutions. Seven parameters transformation model requires mathematically 3 data points for the solution. Relatively small differences in the results might came from the uncertainties in the used actual data points. GPS and traditional observations are not free error and their errors are reflected in the computed residuals.

## 4-1-2 Statistics of the Results at the Actual Check Points (Group 1 Solutions)

The residuals at the five actual check points are computed. Residuals of latitudes and longitudes as well as their resultants are computed. Statistics of those residuals are also computed and tabulated as follows:

**Latitude statistics:**

Table(4) Statistics of latitude residuals for group 1 solutions (actual check points), in m

| | Sol.(1) 10 pts | Sol.(3) 11 pts | Sol.(4) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 2.11 | 2.21 | 2.24 | 1.93 |
| Mean | 0.42 | 0.44 | 0.49 | 0.37 |
| σ | 0.16 | 0.12 | 0.13 | 0.2 |

The latitude residuals are close to each others, the relatively best results are obtained from Solution 4.

**Longitude residuals:**

Table(5)Statistics of longitude residuals for group1 solutions (actual check points), in m

| | Sol.(1) 10 pts | Sol.(2) 11 pts | Sol.(3) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 3.64 | 3.54 | 3.46 | 3.62 |
| Mean | 0.72 | 0.70 | 0.69 | 0.72 |
| σ | 1.08 | 1.06 | 1.08 | 1.14 |

The longitude residuals have very much close values to each others, the best results are obtained from Solution 3 with small differences with the other solutions.

**Resultant residuals:**

Table (6) Statistics of residual resultants for group1 solutions (actual check points), in m

| | Sol.(1) 10 pts | Sol.(2) 11 pts | Sol.(3) 12 pts | Sol.(4) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 4.82 | 4.79 | 4.79 | 4.75 |
| Mean | 0.96 | 0.95 | 0.95 | 0.95 |
| σ | 0.95 | 0.92 | 0.93 | 1.02 |

The different solutions are equal in their resultant statistics. The same conclusion of the data points case can be drawn here. The obtained residuals are within the uncertainties of the used GPS and local coordinates.

As a final conclusion from the seven parameters model of transformation, the different solutions are close to each others in both cases at data points and at check points.

Solution 3 was relatively the best at the data points. Recalling that the solutions gave close results to each other because 10 points are already sufficient number of data points for the solutions.

## 4-2 Group 2 Results: Polynomial Solutions Using Actual Data Points

Residual statistics at the actual data and check points will be tabulated in the following two subsections

### 4-2-1 Statistics of the Results at the Actual Data Points (Group 2 Solutions)

The coefficients of the used polynomials are computed using actual data points in group 2 solutions, the same points used in group 1 solutions. Latitude and longitude residuals as well as their resultants are computed at the actual data points. The statistics of those residuals are then computed and collected in the following tables:

**Latitude residuals:**

Table (7) Statistics of latitude residuals from group 2 solutions (actual data points). in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| $\Sigma\|V\|$ | 0.0 | 3.73 | 3.99 | 4.78 |
| Mean | 0.0 | 0.34 | 0.33 | 0.36 |
| σ | 0.0 | 0.31 | 0.31 | 0.31 |

The residuals in latitude have zero values in solution 5. This is because the number of used points is the minimum (necessary) for the used polynomials, i.e. there is no redundancy. The remaining solutions have almost the same results.

**Longitude residuals:**

Table(8)Statistics of longitude residuals from group2 solutions (actual data points). in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| $\Sigma\|V\|$ | 0.0 | 2.01 | 2.54 | 2.8 |
| Mean | 0.0 | 0.18 | 0.21 | 0.21 |
| σ | 0.0 | 0.16 | 0.16 | 0.25 |

The residuals in longitudes have zero values in solution 5. This is because the number of used points is the minimum (necessary) for the used polynomials, i.e. there is no redundancy. Solution 6 gave a little bit better results than the remainder solutions because it has only two redundant equations.

**Resultant residuals:**

Table(9) Statistics of residual resultants from group2 solutions (actual data points), in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 0.0 | 4.35 | 4.96 | 6.47 |
| Mean | 0.0 | 0.39 | 0.41 | 0.49 |
| σ | 0.0 | 0.35 | 0.34 | 0.36 |

The resultant residuals have zero values in solution 5. The best results are obtained in solution 6 with not big difference with the other solutions. The uncertainties in the added actual data points affect the residuals, they are not error free.

### 4-2-2 Statistics of the Results at the Actual Check Points (Group 2 Solutions)

Group 2 solutions are examined at five actual check points, same points used in group 1 solutions, and the residuals at those points as well as their statistics are computed and the statistics of those residuals are tabulated as follows:

**Latitude residuals:**

Table (10) Statistics of latitude residuals from group2 solutions (actual check points), in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 26.13 | 0.86 | 0.72 | 1.23 |
| Mean | 5.22 | 0.17 | 0.14 | 0.24 |
| Σ | 4.98 | 0.09 | 0.09 | 0.49 |

Solution 5 has big residuals values compared to the other solutions. Data points in solution 5 are minimum (necessary) to the polynomials solution. The used polynomials fitted the used 10 data points but it did not express the whole area very well, it still needs densification. So, the residuals are dramatically reduced by increasing one data point to the minimum number of data points. Solution 7 gave better results among the remainder solutions. The data used are actual coordinates and they are not error free, this affects the residuals according to the errors in those data points.

**Longitude residuals:**

Table (11) Statistics of longitude residuals from group2 solutions (actual check points)in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 13.33 | 3.70 | 4.03 | 4.37 |
| Mean | 2.66 | 0.74 | 0.80 | 0.87 |
| σ | 2.80 | 0.98 | 0.97 | 0.48 |

Again solution 5 has big residuals values compared to the other solutions. Again the polynomials need densification. So, the residuals are dramatically reduced by increasing one data point to the minimum number of data points. Solution 6 gave better results among the remainder solutions. The data used are actual coordinates and they are not error free, this affects the residuals according to the errors in those data points.

**Resultant residuals:**

Table (12) Statistics of residual resultants from group 2 solutions (actual check points) in m

| | Sol.(5) 10 pts | Sol.(6) 11 pts | Sol.(7) 12 pts | Sol.(8) 13 pts |
|---|---|---|---|---|
| Σ\|V\| | 29.37 | 6.88 | 4.19 | 4.69 |
| Mean | 5.87 | -1.37 | 0.83 | 0.93 |
| σ | 5.73 | 1.15 | 0.46 | 0.92 |

Again solution 5 has big residuals values compared to the other solutions, 10 points are mathematically sufficient to the solution but they are not dense enough for good representation. The residuals are dramatically reduced by increasing one data point to the minimum number of data points. Solution 7 gave better results among the remainder solutions. The data used are actual coordinates and they are not error free, this affects the residuals according to the errors in those data points.

## 4-3 Comparison Between Molodenski and the Used Polynomials Solutions

The statistics of the resultants from the solutions of group 1 and group 2 are collected here for the sake of comparison. The statistics are collected and tabulated once for the data points and another time for the check points.

**Data points comparison:**

Table (13): Resultant statistics at actual data points of group1 and group 2

| | 10 points sol. | | 11 points sol. | | 12 points sol. | | 13 points sol. | |
|---|---|---|---|---|---|---|---|---|
| | Mol. | Poly. | Mol. | Poly. | Mol. | Poly. | Mol. | Poly. |
| Σ\|V\| | 14.6 | 0.0 | 14.7 | 4.35 | 15.47 | 4.96 | 17.26 | 6.47 |
| Mean | 1.46 | 0.0 | 1.33 | 0.39 | 1.28 | 0.41 | 1.32 | 0.49 |
| σ | 0.79 | 0.0 | 0.48 | 0.35 | 0.81 | 0.34 | 0.76 | 0.36 |

The polynomials fit the used 10 data points exactly because they are the minimum number of points to be used for the solution. In the other three categories of solutions using 11, 12, and 13 data points, the polynomials gave better results than Molodenski solutions.

**Check points comparison:**

Table (14) Resultant statistics at actual check points of group1 and group 2

| | 10 points sol. | | 11 points sol. | | 12 points sol. | | 13 points sol. | |
|---|---|---|---|---|---|---|---|---|
| | Mol. | Poly. | Mol. | Poly. | Mol. | Poly. | Mol. | Poly. |
| Σ\|V\| | 4.82 | 29.37 | 4.79 | 6.88 | 4.79 | 4.19 | 4.75 | 4.69 |
| Mean | 0.96 | 5.87 | 0.95 | 1.37 | 0.95 | 0.83 | 0.95 | 0.93 |
| σ | 0.95 | 5.73 | 0.92 | 1.15 | 0.93 | 0.46 | 1.02 | 0.92 |

In 10 points solutions, Molodenski is much better than the used polynomial. There is 7 redundant points with respect to Molodenski model while there is no redundancy in the polynomial solution. One redundant point in the polynomial improved its results considerably but Molodenski still better. Using 12 data points improved the polynomial to be better than Moldenski, i.e the polynomial is better with 2 redundant points and average spacing 295 km

## 4-4 Group 3 Results: Polynomial Solutions Using Simulated Grid Data Points

Five solutions, from solution 9 to solution 13, are made using simulated grid data points starting with 10 points as minimum and ending with 14 points. In all five solutions, the residuals at the data points in both latitude and longitude as well as the resultant, have zero values. This means that in all solutions the polynomials fit the data points exactly, the used data points here are assumed, i.e. they are error free. So, there is no need to tabulate the results of those solutions.

The residuals at the actual check points as well as the simulated check points are computed for every solution, and they are tabulated as follows:

### 4-4-1 Statistics of the Results at the Actual Check Points (Group 3 Solutions)

**Latitude residuals:**

Table (15) Statistics of latitude residuals from group 3 solutions (actual check points), in m

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 1488.89 | 1636.04 | 133.79 | 2.55 | 2.55 |
| Mean | 297.78 | 327.21 | 26.76 | 0.51 | 0.51 |
| σ | 105.78 | 116.19 | 9.91 | 0.26 | 0.26 |

The first three solutions have mathematically sufficient data points but those data points are not dense enough to represent the whole area of Egypt. So, the residuals in solution 12 are dramatically reduced. Solutions 12 and 13 are identical, i.e 13 data points are enough.

**Longitude residuals:**

Table (16) Statistics of longitude residuals from group 3 solutions (actual check points), in m

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 1329.6 | 6063.55 | 1956.35 | 6.39 | 6.39 |
| Mean | 265.91 | 1212.71 | 391.27 | 1.27 | 1.27 |
| σ | 95.71 | 427.42 | 136.8 | 0.98 | 0.98 |

On the contrary of the latitude case, solution using 10 data points is better than 11 points solution, the distribution of the eleven used data points was not good. Residuals in solutions 12 and 13 are identical and dramatically reduced because 13 data points are enough for good representation in the whole area of Egypt.

**Resultant residuals**

Table (17) Statistics of resultant residuals from group 3 solutions (actual check points), in m

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 1996.00 | 6280.39 | 1960.92 | 7.46 | 7.46 |
| Mean | 399.22 | 1256.07 | 392.18 | 1.49 | 1.49 |
| σ | 142.65 | 442.92 | 137.15 | 0.79 | 0.79 |

Again solutions 12 and 13 are identical and dramatically reduced residuals because data points less than 13 is not enough for good representation allover the Egyptian territory.

### 4-4-2 Statistics of the Results at the Simulated Check Points (Group 3 Solutions)

**Latitude residuals:**

Table(18) Statistics of latitude residuals from group 3 solutions (simulated check points), in m

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 4936.67 | 5424.54 | 443.54 | 0.00496 | 0.00517 |
| Mean | 274.25 | 301.36 | 24.64 | 0.00027 | 0.00028 |
| σ | 89.49 | 98.33 | 8.04 | 0.00019 | 0.000144 |

Latitude residuals at the simulated check points have the same behavior of their corresponding residuals at the actual check points. Solutions using 12 and 13 data points have identical zero residual.

**Longitude residuals:**

**Table (19) Statistics of longitude residuals from group 3 solutions (simulated check points), in m**

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 4425.94 | 20087.38 | 6469.12 | 0.011 | 0.009 |
| Mean | 245.88 | 1115.96 | 359.39 | 0.000 | 0.000 |
| σ | 80.23 | 364.14 | 117.27 | 0.000 | 0.000 |

Longitude residuals at the simulated check points have the same behavior of their corresponding residuals at the actual check points. Solutions using 12 and 13 data points have identical zero residual.

**Resultant residuals:**

Table (20) Statistics of resultant residuals from group 3 solutions (simulated check points), in m

| | Sol.(9) 10 pts | Sol.(10) 11 pts | Sol.(11) 12 pts | Sol.(12) 13 pts | Sol.(13) 14 pts |
|---|---|---|---|---|---|
| Σ\|V\| | 6630.21 | 20806.93 | 6484.31 | 0.012 | 0.012 |
| Mean | 368.34 | 1155.94 | 360.24 | 0.000 | 0.000 |
| σ | 120.19 | 377.18 | 117.54 | 0.000 | 0.000 |

Residuals of 11 data points solution are much bigger than those of 10 points solution, the distribution of the used 11 data points is not good. Using 12 data points improves the results. Solutions using 12 and 13 data points have identical zero values.

In the above three tables, the first three solutions have mathematically enough data points but those points are not enough for good densification in the whole area of Egypt. Thirteen points with their distribution in Figure (5) are found to be sufficient for good representation. This was also the case of using actual check points.

Solution using 13 data points gives the minimum residuals, the best solution. It gives minimum residuals in the case of the actual check points, the coordinates of the check points are not error free. The solution gives zero residuals at the simulated check points because those points are error free (simulated). Recalling that solution 12 used 13 data points with 485 km average spacing. Increasing data points and decreasing the average spacing to improve the polynomials solution is useless.

## 5- Conclusion

This research is made to investigate the behavior of the polynomial in the transformation of coordinates process. The polynomial is investigated against the number of used data points and the average spacing among those points. Group 1 solutions are made using the classic Molodenski 7 parameters model. Group 1 solutions are made to be base for evaluating the results of the adopted polynomial solutions in gruop2. Group 1 and group 2 solutions are computed using the same scattered actual data points. Group 3 solutions are computed using simulated grid data points. All solutions are checked using actual and simulated check points.

Considering the Egyptian territory, the following can be concluded:

- Creating the polynomial using minimum data points exactly fits the data points, while it causes large distortions at the check points.
- Optimum using of the polynomial gives better results than using the classic 7 parameters Molodenski model at data and check points.
- When the polynomials are created using actual data points, adding one data point to the required minimum data points makes the polynomials fit the data points better than Molodenski model.
- When the polynomials are created using actual data points, adding two data points to the required minimum data points makes the polynomial better than Molodenski model at the check points. This happens when 12 actual data points are used in creating the polynomial with average spacing 295 km.
- When the polynomials are created using 13 simulated grid data points with average spacing 485 km, the minimum residuals at the actual check points are obtained.
- When the polynomials are created using 13 simulated grid data points with average spacing 485 km, zero residuals at the simulated check points are obtained.

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